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Book Review: *Differential Geometry, Gauge Theories, and Gravity*

Differential Geometry, Gauge Theories, and Gravity, M. Göckeler and T. Schücker, Cambridge University Press, New York, 1989

For about the last 20 years modern differential geometry has played a prominent role in theoretical physics. It was Einstein's theory of general relativity that brought differential geometry into the physics community in the early decades of this century, but we had to wait until the success of modern gauge theories to see up-to-date differential geometric tools being employed by a considerable number of physicists. Gauge invariance is also the guiding principle used by M. Göckeler and T. Schücker in the writing of their book. It is intended as an introduction to the concepts of differential geometry used in gauge theories and general relativity on the level of a graduate textbook in physics.

The book consists of 12 chapters. Following each chapter is a problem section to fill in omitted proofs or to broaden the reader's understanding of the presented material. The first two chapters introduce the basic notions of differential geometry with emphasis on differential forms on open subsets of $\mathbb{R}^n$. Chapter three goes on to introduce metrical structures, the hodge star, and holonomic and orthonormal frames. It ends with a summary of the mathematical notions of the first three chapters, both index-free and with indices, to show the advantages of the index-free notation. Chapter four deals with gauge theories. First electromagnetism is discussed as an example of an Abelian gauge theory, then arbitrary gauge groups are treated. The chapter ends with a section on lattice gauge theory and a summary. Chapter five covers general relativity, or, to be specific, Einstein–Cartan theory. This is done in a manner that makes it look like a gauge theory, although the discussion fails to emphasize the important differences. The derivation of the Einstein–Cartan field equations shows nicely the economy of the differential form calculus. The authors treat differential geometry again in the next two chapters with the Lie derivative and the introduction of manifolds. Lie groups and Lie algebras are introduced in chapter eight. Fiber bundles are explained at length in chapter nine. The next chapter

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shows applications of fiber bundle techniques, namely monopoles and instantons, as well as a treatment of Chern classes. Chapter eleven is devoted to spin. The subject is described entirely in terms of Clifford algebras and their related groups, followed by sections on spin structures, the Dirac operator, and Kähler fermions. The last two chapters focus upon the subject of infinitesimal anomalies, beginning with the algebraic approach to anomalies as solutions to the Wess-Zumino consistency condition and followed by a calculation of anomalies via Feynman graphs.

In 230 pages the authors compress a large amount of material and it is therefore not surprising that most of the topics do not get covered in depth. Some sections should have been either expanded or treated as an aside; for example, the section on lattice gauge theory consists of only 13 lines. Because of the authors' concise and lucid style of writing, the book is well suited as a handy reference work for both mathematicians and physicists. It can also serve as a sketchy introductory course for physics graduate students, although they should already be acquainted with gauge theories and general relativity. This book is not, however, a conventional textbook on the physical applications of differential geometry, because it covers areas well out of the ordinary. This will make it interesting to a wide audience. The book is also available in an inexpensive paperback edition.

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